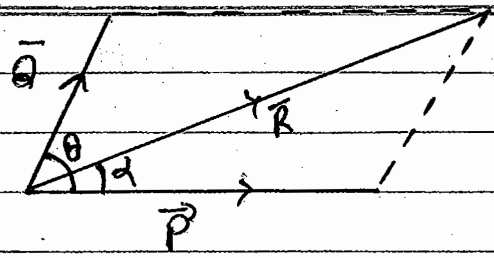
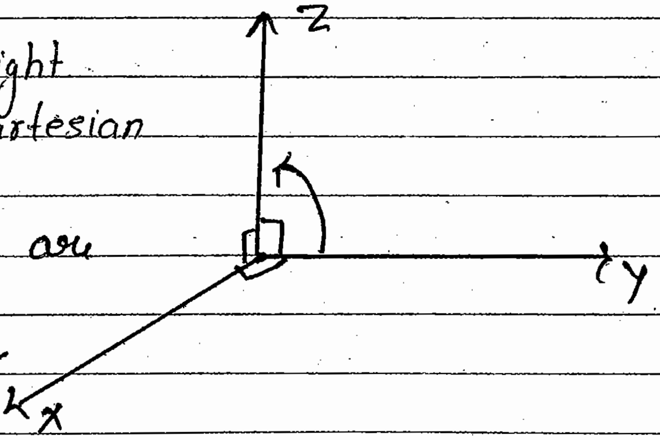


$$\boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}}$$

$$\boxed{\sin\alpha = \frac{Q\sin\theta}{R}}$$

* Co-Ordinate System :-

1. Right Handed Cartesian System :-



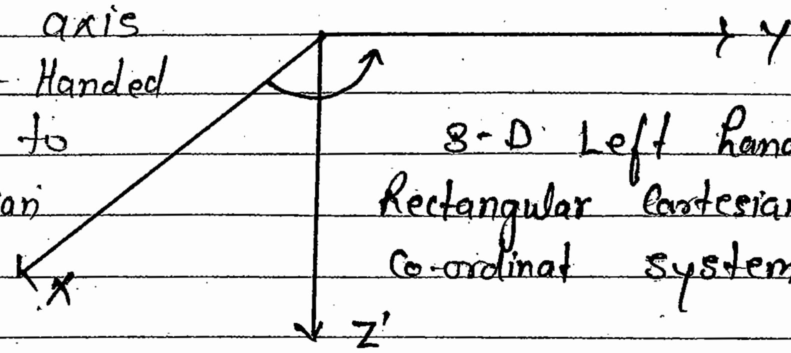
⇒ 3-D ~~Rectangul~~ Right handed Rectangular cartesian coordinate system.

⇒ Here x, y, z axis are mutually perpendicular so it is rectangular

⇒ The orientation of x, y, z axis follow Right Handed Thumb Rule, so it is Right Handed.

2. Left Handed Co-ordinate system :-

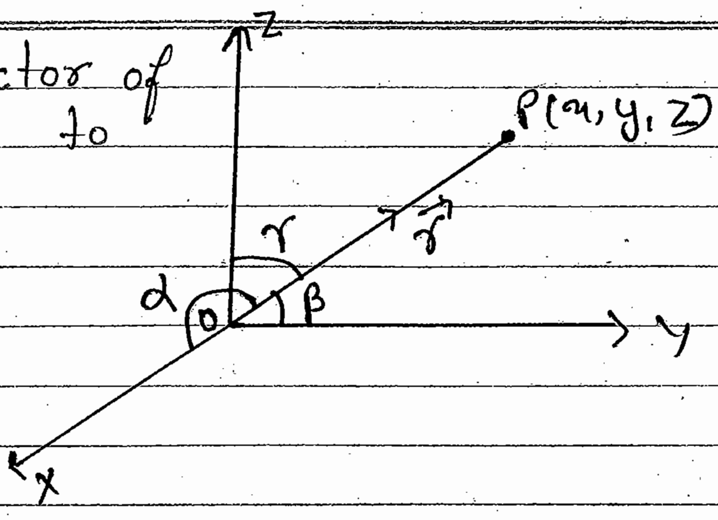
Invert any one axis to convert Right Handed Cartesian system to Left Handed cartesian System.



3-D Left handed Rectangular cartesian Co-ordinat system.

→ It follow Left Hand Thumb Rule.

$\vec{OP} = \vec{r}$ = Position Vector of point P with respect to origin O.



$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\boxed{r = \sqrt{x^2 + y^2 + z^2}}$$

Where $\hat{i}, \hat{j}, \hat{k} \rightarrow$ Orthogonal basis

Note:-

$r = \sqrt{x^2+y^2+z^2}$, This formula is applicable only when $\hat{i}, \hat{j}, \hat{k}$ are mutually perpendicular.

Unit Vector Along $\vec{r}$:-

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\vec{r}}{r}$$

So

$$\boxed{\hat{r} = \frac{x}{r}\hat{i} + \frac{y}{r}\hat{j} + \frac{z}{r}\hat{k}}$$

* Direction Cosines of $\vec{r}$:-

If α, β, γ is the angle, which $\vec{r}$ making with x, y, z axis, then -

$$l = \cos\alpha = \frac{x}{r}$$

$$m = \cos\beta = y/r$$

$$n = \cos\gamma = z/r$$

$$\boxed{l^2 + m^2 + n^2 = 1}$$

So $\boxed{\hat{r} = l\hat{i} + m\hat{j} + n\hat{k}}$

$$\boxed{\hat{r} = \cos\alpha\,\hat{i} + \cos\beta\,\hat{j} + \cos\gamma\,\hat{k}}$$

Where l, m, n are the direction cosines.

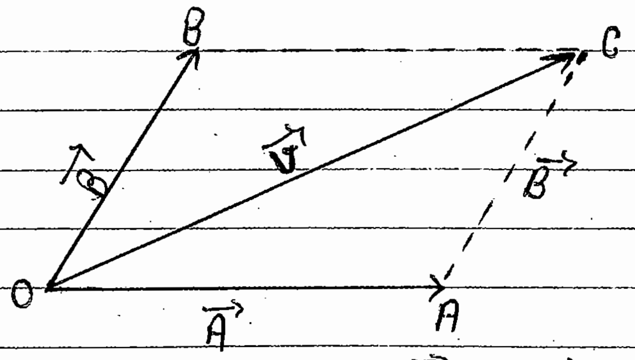
* Addition of Vectors :-

The composition of two displacements of a point has all the characteristics of a summation.

Let us consider two vectors $\vec{A}$ and $\vec{B}$ acting on a point O as shown in figure (a). The resultant effect of the vectors $\vec{A}$ and $\vec{B}$ is same as that the their vector sum $\vec{V}$. The vector $\vec{V}$ is obtained by setting off the vector $\vec{B}$ at the end of $\vec{A}$ and drawing the vector $\vec{V}$ joining the begining of $\vec{A}$ to the end of $\vec{B}$.

So $\vec{V} = \vec{OC} = \vec{OA} + \vec{AC}$

$$\boxed{\vec{V} = \vec{A} + \vec{B}}$$



A similar result is obtained by starting with $\vec{B}$ and setting of the vector $\vec{A}$ on $\vec{B}$.

So,

$$\vec{V} = \vec{OC} = \vec{OB} + \vec{BC} = \vec{B} + \vec{A}$$

$$\boxed{\vec{A} + \vec{B} = \vec{B} + \vec{A}}$$

Which shows that the sum of two vectors is the diagonal of the parallelogram with the vectors as its sides and this sum obeys the law of commutation according to which the sum of two quantities is independent of the order of the quantities.

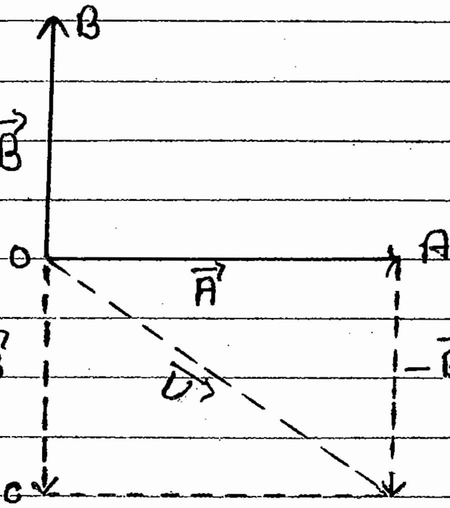
* Subtraction of Vectors:-

The sum of two vectors is zero when and only when they have the same lengths but opposite directions,

$$\vec{A} + \vec{B} = 0$$

When,

$$\vec{A} = -\vec{B}$$



Thus the negative of a vector is a vector of the same length but opposite direction $-\vec{B}$

So,

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

Which shows that subtracting a vector $\vec{B}$ from vector $\vec{A}$ is same as adding its negative.

* Product of Vectors:-

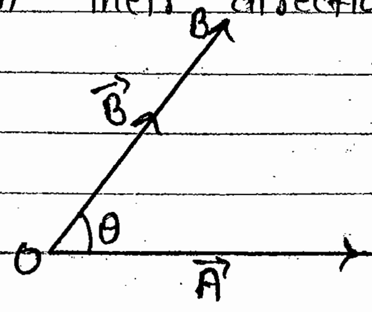
Two vectors are ~~multiplied~~ multiplied in two manners, i.e. scalarly and vectorily. These products are discussed seperately.

(a) The Scalar Product

The scalar product of two vectors $\vec{A}$ and $\vec{B}$ is defined as the product of the magnitudes of vectors and the cosine of the angle between their directions,

Thus,

$$\vec{A} \cdot \vec{B} = AB\cos\theta = OA \cdot OB\cos\theta$$



Thus, scalar product of two vectors is the product of

the size of one vector with the component of the other in the direction of the first.
Moreover,

$$\vec{B}.\vec{A} = BA\cos(-\theta) = BA\cos\theta = A.B$$

Hence the scalar product of two vectors is commutative, i.e. independent of the order of vectors.

$$\because \quad \vec{A}.\vec{B} = |\vec{A}||\vec{B}|\cos\theta$$

Such that if -

$$\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$
$$\vec{B} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

Then

$$\vec{A}.\vec{B} = a_1b_1 + a_2b_2 + a_3b_3$$

$$\vec{A}.\vec{B} = \vec{B}.\vec{A}$$

Properties of $\hat{i}, \hat{j}, \hat{k}$:-

$$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} \ldots\ldots = 1$$
$$\hat{i}.\hat{j} = \hat{j}.\hat{k} = \ldots\ldots = 0$$

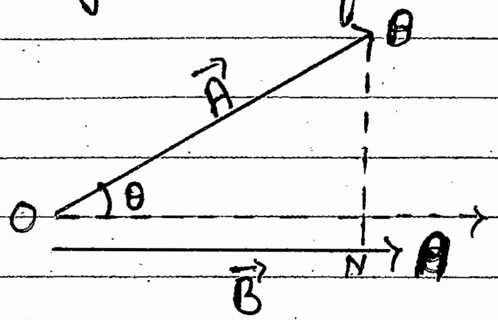
$$\langle \psi_1 | \psi_2 \rangle = 1 \longrightarrow \text{Normalization}$$

$$\langle \psi_1 | \psi_2 \rangle = 0 \longrightarrow \text{Orthogonal}$$

* Application of Dot Product :-

(i) $$\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}$$

(2) Projection of $\vec{A}$ on $\vec{B}$



Complea of $\vec{A}$ along $\vec{B}$

$= \vec{A} \cdot \hat{B}$

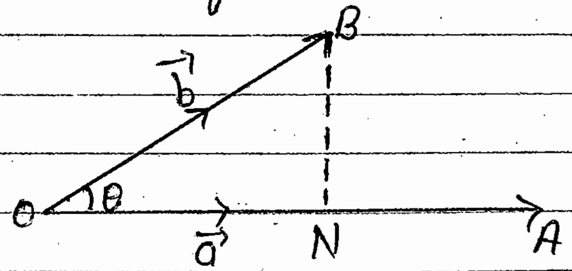
$= |\vec{A}|\,|\hat{B}|\,\cos\theta$

$= A\cos\theta$

OR.

The scalar product of two vectors is the product of one vector and the length of the projection of the other in the direction of the first.

$\vec{OA} = \vec{a}$, $\vec{OB} = \vec{b}$



then,

$\vec{a}\cdot\vec{b} = (OA).(OB)\cos\theta$

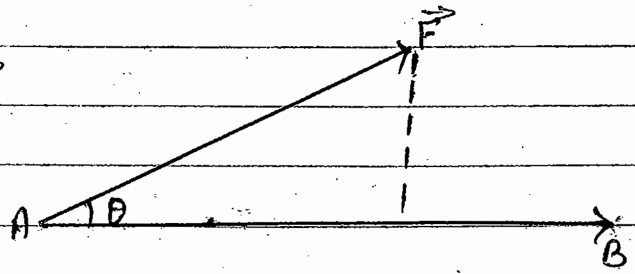
$= (OA).(OB).\dfrac{(ON)}{(OB)}$

$= (OA).(ON)$

$=$ (Length of $\vec{a}$)(projection of $\vec{b}$ along $\vec{a}$).

(3) Work done as a Scalar Product :-

If a constant force $\vec{F}$ acting on a particle displaces it from A to B then,



Workdone $=$ (Component of $\vec{F}$ along AB). Displacement

$= F\cos\theta . AB$

$= \vec{F}.\vec{AB}$

Workdone $=$ Force . Displacement.